A rapid financial seismic risk assessment methodology with application to bridge piers

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ABSTRACT: Expected annual loss (EAL), is an effective way of communicating the seismic vulnerability of constructed facilities to decision makers. A simplified method for approximating EAL without conducting time consuming non-linear dynamic analyses is presented. Relationships between *intensity measures* and *engineering demand parameters* resulting from a pushover analysis and a modified version of the capacity spectrum method are combined with a variety of epistemic and aleotoric uncertainties to arrive at a demand model. Financial implications due to damage are quantified by loss ratios defined as repair cost divided by replacement cost. The method is verified by performing a comprehensive *Incremental Dynamic Analysis*. An example illustrating the method is performed, comparing the seismic vulnerability of two highway bridge piers; one designed for ductility, and the other designed for damage avoidance. The damage avoidance pier has a clear advantage over the conventional pier, with an EAL some 80% less than its ductile counterpart.

1 INTRODUCTION

One primary aim of *Performance Based Earthquake Engineering* (PBEE) is to predict, with a certain level of confidence, the seismic performance of structures at various levels of earthquake excitation (seismic demand). This requires the engineer to understand seismic risk and its inherent uncertainty. As an adjunct to conventional design it is desirable that the engineer be able to communicate that risk in a way easily understood by stake-holders such as owners, bankers, and insurers. One primary development is the *Pacific Earthquake Engineering Research* (PEER) *Center's* triple integral framework equation, which can be used to arrive at a mean annual frequency of a decision variable (Deierlein et al. 2003). This equation can be broken into four subtasks: (i) assessment of seismic hazard; (ii) analysis for structural response; (iii) quantification of damage; and (iv) estimation of the decision variable.

Der Kiureghian (2005) has hinted on the possibility of performing a fourth integration, thereby arriving at the mean cumulative value of a *decision variable* (DV) given one year in time. In the context of financial risk assessment, this could be expressed in the form of *expected annual loss* (EAL). EAL is an effective communication tool as it incorporates a range of seismic scenarios, return rate, and expected damage into a single median dollar loss. Dhakal & Mander (2006) have followed such an approach.

A primary step within PBEE is defining a relationship between specified demand levels and a hazard environment. This relationship, termed the demand model, has gained a lot of attention in the past decade. Vamvatsikos and Cornell (2002) have researched the feasibility of *Incremental Dynamic Analysis* (IDA) as a means of relating these parameters. An IDA basically consists of performing a series of time-history analyses to arrive at a set of demand parameters, obtained by scaling a suite of earthquake records to various intensities. From this analysis, further data processing will yield a probabilistic relationship between *engineering demand parameters* (EDP) and *intensity measures* (IM). This forms the theoretical backbone of subsequent models relating structural response to damage and damage to loss

This paper will set out to estimate EAL using a rapid analysis approach, referred to hereafter as the *Rapid IDA-EAL* approach. This approach will approximate the median IDA curve through relationships between demand and capacity and use assumed probability distributions to estimate EAL through simplified formulae. A case study of two different bridge piers will illustrate the effectiveness of the approach for estimating EAL. The Rapid IDA-EAL method will be verified by a rigorous computational IDA method.

2 RAPID IDA-EAL THEORY

In a general sense, EAL can be calculated by an extension of the PEER framework equation:

$$E[L_r] = \int\int\int\int L_r dG(L_r \mid dm) \mid dG(dm \mid edp) \parallel dG(edp \mid im) \parallel d\lambda(im) \mid \quad (1)$$

where *im* = intensity measure (i.e. peak ground acceleration, spectral acceleration); *edp* = engineering demand parameter (i.e. maximum interstory drift); *dm* = damage measure (i.e. maximum drift without damage); L_r = loss ratio defined as the cost to repair a structure divided by the total replacement cost; and $G(x|y) = P(x<X|Y=y)$; the conditional complementary cumulative distribution function. This can be described graphically by the hazard-survival curves given in Figure 1; the volume of this plot will yield EAL.

Using the Rapid IDA-EAL method, it is possible to generate the curves given in Figure 1. The concept of the proposed method is relatively straightforward. It is possible to generate the median IM versus EDP relationship from a non-linear *static pushover* (SPO) analysis and a modified *capacity spectrum method* (CSM). Adopting the customary assumption that variability conforms to a lognormal distribution (Mander & Basoz 1999), fragility curves can be generated for discrete states of damage. The fragility curves are then transformed via a hazard-recurrence relationship into hazard-survival curves for each damage state. Then, financial implications of the different damage states are considered together with the corresponding hazard-survival curves to arrive at the EAL. This general process is outlined in detail through the following steps.

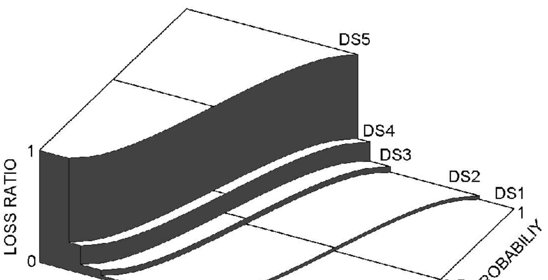


Figure 1. Hazard-survival curves plotted with each associated loss ratio.

2.1 *Step 1: Conduct Pushover Analysis*

A non-linear SPO analysis is performed to assess the capacity of the system, as given in Figure 2. From the SPO curve, it is possible to calculate the secant (equivalent elastic) period T in terms of normalized base shear capacity C_c and peak response displacement Δ as follows:

$$T = 2\pi\sqrt{\frac{M}{K}} = 2\pi\sqrt{\frac{W}{g}\frac{\Delta}{C_c W}} = 2\pi\sqrt{\frac{\Delta}{C_c g}} \quad (2)$$

in which C_c can be expanded as $C_c = F_y / W$ where F_y = base shear force; W = seismic weight; M = seismic mass, K = initial stiffness; and g = acceleration of gravity.

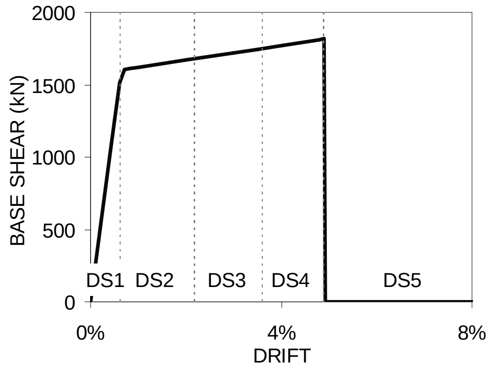


Figure 2. A non-linear static pushover curve.

2.2 *Step 2:Calculate Median IDA Curve*

The evaluation of seismic demand at various effective damping levels depends on the portion of the spectrum governing response. Figure 3 illustrates the seismic demand spectrum and the regions of constant spectral acceleration, spectral velocity, and spectral displacement as limited by T_a, T_v, and T_d. For a given effective (secant) period of vibration T, the normalized base shear demand C_d can be calculated as the lesser of:

$$C_d = \min\left[\frac{F_a S_s}{B_a}; \frac{F_v S_1}{T B_v}; \frac{F_v S_1 T_d}{T^2 B_d}\right] \quad (3)$$

where F_a and F_v are factors to adjust spectral acceleration for short and long period structures at different soil classes; S_s and S_1 are spectral acceleration at short periods and the one second period; and B_a, B_v, and B_d are factors based on effective viscous damping for the constant spectral acceleration, velocity, and displacement regions, respectively.

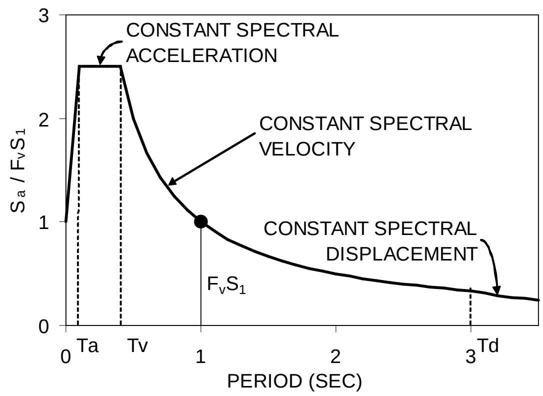


Figure 3. Standard spectral acceleration relationships.

Employing the CSM, it is possible to relate the capacity-displacement curve (SPO curve) and the *acceleration-displacement response spectrum* (ADRS) curve by combining them into a single plot as illustrated in Figure 4. The “performance point” of the structure is estimated from the intersection of the SPO curve with the damping-reduced ADRS curve.

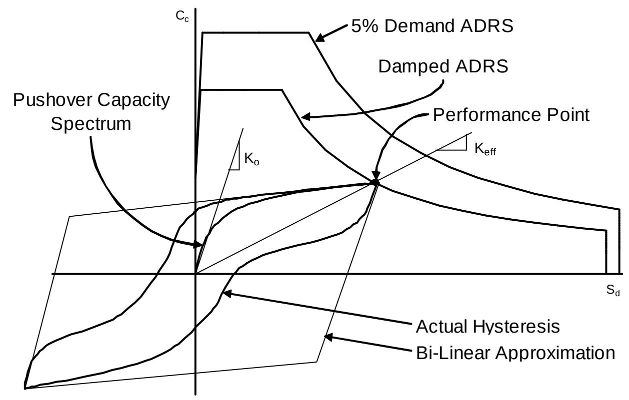


Figure 4. The capacity spectrum method (Pekcan et al. 1999).

The CSM, as presented in ATC-40 (1996), has come under considerable scrutiny due to inconsistent displacement predictions (Miranda et al. 2002). To further address the issue, this study has adopted modified damping approximations proposed by Lin & Chang (2004) coupled with the reduction in equivalent viscous damping due to the pinched nature of the real hysteresis curves, as introduced by Pekcan et al. (1999). Based on recent studies by Lin & Chang (2004) and modified herein as part of the present study, the damping-related reduction factors, B_a, B_v, and B_d can be calculated as a function of effective damping, ξ_{eff} as follows:

$$B_a = \sqrt{\frac{2+\xi_{eff}}{7}} \tag{4}$$

$$B_d = \sqrt{\frac{8+\xi_{eff}}{13}} \tag{5}$$

The damping factor for the constant spectral velocity range, B_v can be calculated by linear interpolation between B_a and B_d based on period or spectral displacement. Total effective viscous damping can be estimated by using the method proposed by Pekcan et al. (1999):

$$\xi_{eff} = \xi_o + \xi_{hy} = \xi_o + \frac{2}{\pi}\eta\frac{(1-\alpha_s)(1-1/\mu)}{(1-\alpha_s+\mu\cdot\alpha_s)} \tag{6}$$

in which ξ_o = intrinsic damping of an elastic system; η = the efficiency factor defined as the ratio of the actual area within a hysteresis loop to that of the idealized bi-linear loop; α_s = post-yield stiffness to initial stiffness ratio; $\mu = \Delta_{max} / \Delta_{yield}$ where Δ = displacement at the seismic centre of mass of the structure.

Setting C_c=C_d and substituting Equation (2) into Equation (3), the one second spectral acceleration *(F_vS_1)* for a given demand can be found. Thus for the median IDA curve shown in Figure 5, the IM (spectral acceleration) can be found for a given EDP (displacement) by the *greater* of the following three equations:

$$F_vS_1 = T_v \cdot B_a \cdot C_c \tag{7}$$

$$F_vS_1 = 2\pi\sqrt{\frac{C_c\cdot\Delta}{g}}\cdot B_v \tag{8}$$

$$F_vS_1 = \frac{4\pi^2}{g\cdot T_o}\cdot\Delta\cdot B_d \tag{9}$$

where generally, T_v, and T_d can be taken as 0.4 and 3.0 seconds, respectively.

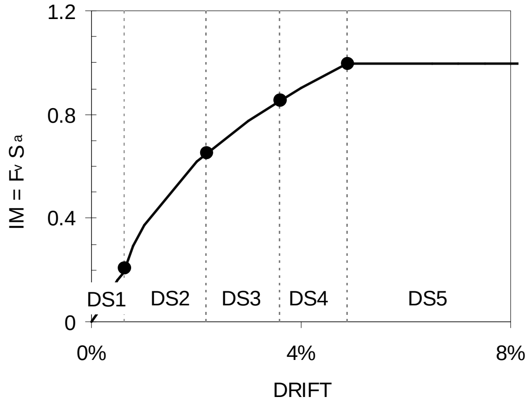


Figure 5. Median IDA curve calculated from the Rapid IDA-EAL method.

2.3 *Step 3: Define Damage States and Limits*

This study adopts the five *damage states* (DS1 to DS5) defined by Mander & Basoz (1999) that have been used in *Hazus*, as summarized in Table 1.

Table 1. Damage states index according to *Hazus*.

Damage State		Repair Required	Outage
DS1	None	None	None
DS2	Minor	Inspect, Adjust, Patch	< 3 days
DS3	Moderate	Repair Components	< 3 weeks
DS4	Major	Rebuild Components	< 3 months
DS5	Complete	Rebuild Structure	> 3 months

2.4 *Step 4: Incorporate Sources of Variation using Assumed Distributions*

Throughout this process numerous approximations are made regarding damping, material strengths, modelling simplifications, etc. These approximations can be grouped into epistemic uncertainty, where further investigation may lead to an increase in accuracy, and aleotoric variability (randomness), which cannot be reduced because of its random nature. An example of the former would be uncertainty in analytical modelling, and the latter would be the inherent record-to-record randomness of earthquake ground motions. As discussed earlier, previous studies have shown that these variations approximately conform to a lognormal distribution. This two-parameter distribution can be defined with a median ($\tilde{x}$) and a lognormal standard deviation, β, referred to herein as the dispersion factor. Since a formula relating EDP and IM is available, the median values have been established and only the dispersion is left to be determined. It is possible to assume a dispersion based on established trends regarding the various uncertainties discussed. To determine the dispersion of all combined uncertainty and randomness, they are combined by root-sum-squares method established by Kennedy et al. (1980):

$$\beta_{C/D} = \sqrt{\beta_D^{\,2} + \beta_C^{\,2} + \beta_U^{\,2}} \qquad (10)$$

where β_D = the variation of structural response due to the input motion, β_C = the aleotoric randomness in structural capacity (usually considered in the damage model), and β_U = epistemic modeling uncertainty. In this study, recommendations of FEMA 350 [7] have been adopted; i.e. β_C = 0.2 and β_U = 0.25. Although β_D is difficult to quantify, it is likely to vary depending on the IM considered. Investigations into the variation of input motion have been conducted by the authors. It was noted that the variation of β_D can be approximated by the relationship:

$$\beta_D = \beta_{DBE}\sqrt{\frac{IM}{IM_{DBE}}} \qquad (11)$$

where β_{DBE} = lognormal standard deviation of the structural response due to ground motions scaled to the design basis earthquake level, IM_{DBE}. Unlike other vulnerability methods, all uncertainty and randomness are grouped into a single composite dispersion factor, $\beta_{C/D}$. This greatly simplifies the subsequent integration.

2.5 *Step 5: Define an Earthquake-Recurrence Relationship*

To arrive at an EAL, it is necessary to define a relationship between an IM and annual frequency (f_a), which is commonly known as the hazard-recurrence relationship. It is possible to approximate the hazard-recurrence curve by fitting a straight line through two known points in a log-log scale:

$$f_a(IM) = k_o (IM)^{-k} \qquad (12)$$

where k_o and k are empirical constants. Using the 1 second spectral acceleration (F_vS_1) as the IM, Figure 6 plots equation (12) which can also be written as follows for a high seismic zone in New Zealand (k = 3):

$$f_a = \frac{1}{475}\left(\frac{F_v S_1}{0.4}\right)^{-3} \qquad (13)$$

As noted by Der Kiureghian (2005), earthquakes are discrete, rather than continuous events, and should strictly be modelled as a Poisson process. In this case, the hazard-recurrence relation formulated above, though conservative, is perhaps not strictly correct when f_a > 0.01 (T < 100 years). The aforementioned deficiency can be rectified by disregarding any damage below a certain threshold. In this paper, this threshold is assumed to correspond to 90% probability of not sustaining any damage. In other words, this is the intersection of the 90th percentile curve and the line serving as the boundary between DS1 (no damage) and DS2 (slight damage).

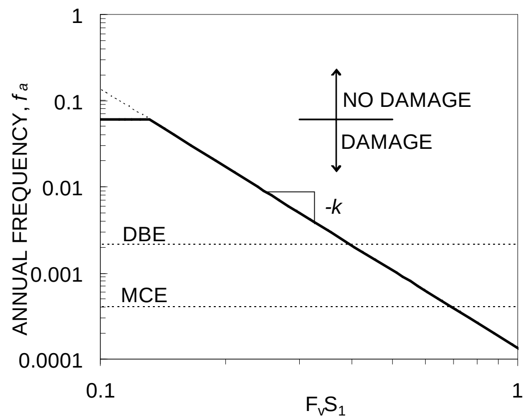


Figure 6. Hazard-recurrence relationship.

2.6 *Step 6: Calculate EAL*

In order to evaluate the EAL, financial implications of the different damage states must be quantified. This is done through a *loss ratio* (LR), which is the ratio of the repair cost to the total replacement cost. Selecting an appropriate LR for each damage state is a subjective process and the accuracy of results will depend largely on the amount of time devoted to researching historical repair costs and their variation with respect to the extent of damage, location of structure, etc.

Hazard-survival curves shown in Figure 1 relate the probability of not exceeding a damage state given an annual frequency, and these curves must be integrated and multiplied with the corresponding LR's to estimate EAL. In other words, EAL is the total volume subtended by the hazard-survival curves for different damage states plotted in the horizontal plane and their corresponding loss ratios plotted in the vertical axis as shown in Figure 1. Using Gaussian quadrature principles, a direct expression for the numerical integration of a cumulative probability curve conforming to a lognormal distribution covering the total probability range (i.e. between 0 and 1) is:

$$EAL = \tilde{x}\left[0.75 + 0.125\left(7^{k\beta} + 7^{-k\beta}\right)\right] \qquad (14)$$

where β = lognormal standard deviation from Equation (10) and k = hazard recurrence parameter defined above. In Equation (14), the median variable $\tilde{x}$ for n damage states is defined as:

$$\tilde{x} = \sum_{i=1}^{n} f_{a_i}\Delta LR_i \qquad (15)$$

where $\Delta LR_i = LR_i - LR_{i-1}$ and f_{a_i} is the annual frequency corresponding to 50% survival probability of the i^{th} damage state boundary. Equation (14) was compared with numerical integration, which showed good agreement with results falling within 1% for $k\beta$ < 2. This formula, however, is conservative and will lead to a higher EAL since it does not consider the cut-off of damage from frequent events. To account for this, Equation (14) can be modified to truncate the data above the 90% no-damage confidence threshold:

$$EAL = \tilde{x}\left[0.6 + 0.2\left(3.5^{k\beta} + 3.5^{-k\beta}\right)\right] \quad (16)$$

Figure 7 illustrates the resulting total loss ratio curve as a function of annual frequency. Taking a single value from this curve gives a scenario loss, similar to what the PEER triple integral equation does. Performing the additional integration yields the EAL, illustrated as the area under the curve (Eq. 1).

To calculate EAL based on the proposed Rapid IDA-EAL, the engineer needs to define only two sets of parameters: the EDP limits for each damage state, and the associated LR's. Once the EAL contributions for each damage state have been calculated using Equations (14) and (16), their summation will give the total EAL for the assumed dispersion. Note that all calculations are based on median values and can be computed by hand. The randomness and uncertainty are combined in a single parameter $\beta_{C/D}$ which is introduced in the process only in the final step. This eliminates difficult integration steps and simplifies the process to such extent that it can be completed in a table.

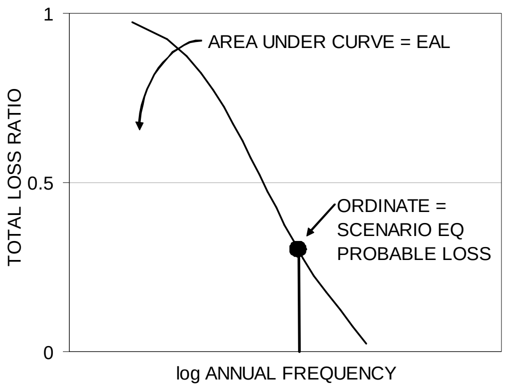


Figure 7. Total loss curve; integrate for EAL.

3 CASE STUDY: RC BRIDGE PIERS

3.1 *Prototype Bridge*

To illustrate the Rapid IDA-EAL method outlined in section 2, consider the reinforced concrete bridge pier shown in Figure 8. The pier is typical of modern design, with 40m spans, 10m transverse width, and 7m height. The seismic weight of the superstructure is assumed to be 7,000kN and it is located in a high seismic zone, on a firm soil site in New Zealand. Consequently, the *peak ground acceleration* (PGA) of the *design basis earthquake* (DBE) with 10% probability in 50 years (i.e. return period of 475 years) is 0.4g and that of the *maximum considered earthquake* (MCE) with 2% probability in 50 years (i.e. return period of 2450 years) is 0.72g. Two structural design alternatives were considered. One alternative being a conventional ductile pier detailed according to the concrete design standard of New Zealand (1995), the second designed according to the principles of *damage avoidance design* (DAD). To avoid damage, the latter implements the steel armouring techniques developed by Mander & Cheng (1997). The bridge column is post-tensioned to the foundation to provide strength and stiffness and supplementary dampers are provided to increase energy dissipation. Both piers were designed for the same base shear capacity.

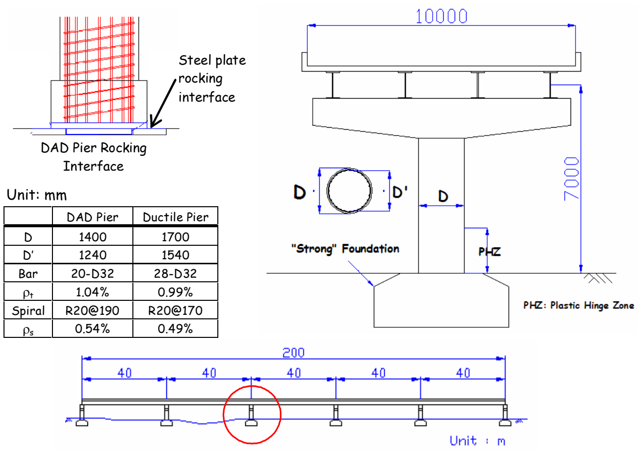


Figure 8. Prototype bridge and design details of the DAD and ductile piers.

3.2 *Expected Annual Loss*

Implementing the Rapid IDA-EAL method, the EAL of the conventional and DAD bridge piers was calculated. The drift limits and loss ratios assigned to the piers is given in Table 2. Loss ratio data assigned to the ductile pier is based on actual cost data given in Mander & Bazos (1999). For both piers, the loss ratio assigned to DS1 was 0. Due to the nature of the DAD pier, it was assumed to remain virtually damage free. However, a 1% allowance was given for yielding of post-tensioned steel for DS2.

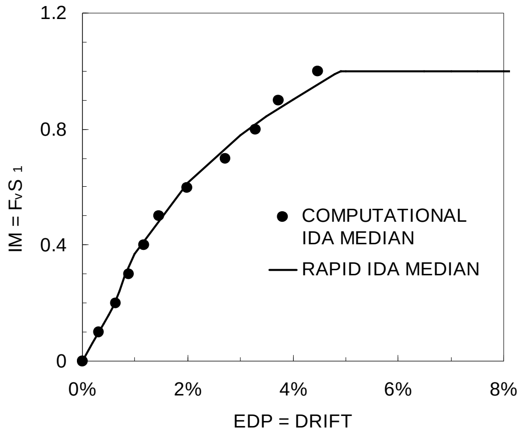
Table 2. Drift limits and loss ratios assigned to the piers.

Damage State	Drift Limits (%)		Loss Ratio (%)	
	Ductile	DAD	Ductile	DAD
DS2	0.6	3.0	3	1
DS3	2.2	-	8	-
DS4	3.6	-	25	-
DS5	4.9	10.0	100	100

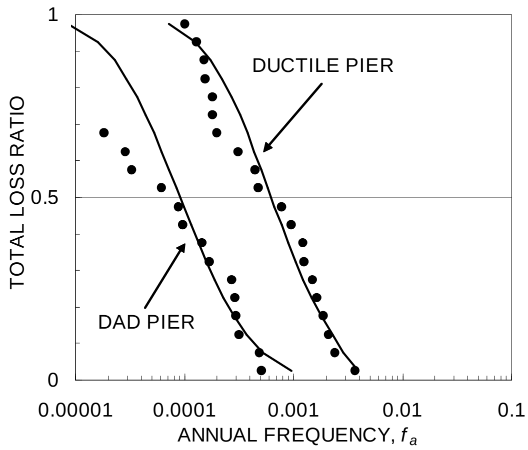
The resulting EAL for the ductile and DAD piers were calculated to be $1,040 and $190 per one million dollars of asset value, respectively.

4 VERIFICATION

To ensure the accuracy of the Rapid IDA-EAL method, results were compared to that of a fully computational IDA-EAL method outlined in Solberg et al. (2006). This procedure was developed to rely on a minimal number of assumptions by performing a large number of inelastic dynamic analyses over a suite of earthquake records using *Incremental Dynamic Analysis* (IDA). The results of the IDA were processed in such a way that the data was sorted into survival probabilities. Figure 9 presents a comparison of the two methods for the ductile bridge pier giving (a) the median IDA curves and (b) the total loss curves. In both cases, the data corresponds well. The EAL calculated by the computational IDA-EAL method for the ductile and DAD piers was calculated to be $990 and $140 per one million of asset value, respectively. For both cases, the resulting EAL differed by only $50 from the Rapid IDA-EAL method.



(a)



(b)

Figure 9. Comparison of the Rapid IDA-EAL method and the computational IDA-EAL method giving (a) median IDA curves for the ductile pier and (b) total loss curves for the DAD and ductile pier.

5 CONCLUSIONS

From this study, the following conclusions can be drawn:

- A rapid method was established to assess financial seismic risk. A non-linear static pushover curve can be combined with the acceleration-displacement response spectrum using the capacity spectrum method to generate the median IDA curve. Given observed trends of data scatter, it can be used to calculate EAL through the use of simplified formulae
- Through the use of the proposed financial risk methods, the seismic vulnerability of two bridge piers with very different detailing was examined. One pier, designed to avoid all forms of damage except from toppling, was shown to have an EAL approximately 80% less than a conventional ductile pier.
- The Rapid IDA-EAL method was compared to a fully computational method. The Rapid IDA-EAL assessment approach showed good agreement with the full computational IDA-EAL approach. Results were well within the same magnitude and varied on average by 15%.

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